CBSE Board
Class XI Mathematics
Sample Paper – 5

Time: 3 hrs   **Total Marks: 100**

General Instructions:
1. All questions are compulsory.
2. The question paper consist of 29 questions.
3. Questions 1 – 4 in Section A are very short answer type questions carrying 1 mark each.
4. Questions 5 – 12 in Section B are short-answer type questions carrying 2 mark each.
5. Questions 13 – 23 in Section C are long-answer I type questions carrying 4 mark each.
6. Questions 24 – 29 in Section D are long-answer type II questions carrying 6 mark each.

SECTION – A

1. In ΔABC, if a = 2, b = 3 and sinA = $\dfrac{2}{3}$, find m∠B.

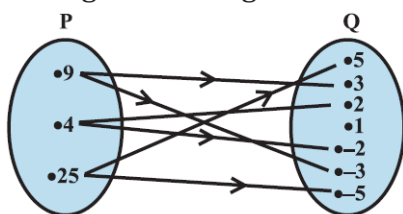**2.** Find the value of $\left(\dfrac{1}{i}\right)^{25}$

OR

Express the $(1 + i)(1 + 2i)$ in the standard form of a + bi.

3. Write the given statement in the form 'If- then', and state what are the component statements p and q.

If I have the money, i will buy an i-phone.

4. The figure below gives a relation. Write it in the roster form.

SECTION – B

5. If P and Q are two sets such that P has 40 elements, P ∪ Q has 60 elements and P ∩ Q has 10 elements, how many elements does Q have?

6. Find the equation of the circle, the co-ordinates of the end points of whose diameter are (-1, 2) and (4, -3).

7. What is the number of ways in which a set of 5 cards can be chosen out of a deck of 52 cards, if each set of 5 cards has exactly one ace?

<div align="center">**OR**</div>

A die is thrown. Find the probability of getting
1. a prime number
2. a number more than 6

8. Prove that $\cos 5x = 16 \cos^5 x - 20 \cos^3 x + 5 \cos x$

9. In how many ways 6 different beads can be arranged on a thread to form a necklace?

<div align="center">**OR**</div>

If $\dfrac{1}{9!} + \dfrac{1}{10!} = \dfrac{x}{11!}$, find x.

10. Find an infinite GP whose first term is 1 and each term is the sum of all the terms which follow it.

11. Find the values of' k' for which $-\dfrac{2}{7}, k, -\dfrac{7}{2}$ are in G.P. Find the common ratio/s of the G.P.

<div align="center">**OR**</div>

Find the sum of 7 terms of the G.P. 3, 6, 12,...

12. If $1 + \dfrac{(1+2)}{2} + \dfrac{(1+2+3)}{3} + ...$ to n terms is S, then find S.

<div align="center">**SECTION – C**</div>

13. Evaluate: $\left| \dfrac{1+i}{1-i} - \dfrac{1-i}{1+i} \right|$

<div align="center">**OR**</div>

If (a + ib) (c + id) (e + if) (g + ih) = A + iB,

then find the value of $A^2 + B^2$

14. Solve the given quadratic equation: $9x^2 - 12x + 20 = 0$

15. Find the co-efficient of x^5 in the expansion of the product $(1 + 2x)^6 (1 - x)^7$

16. Show that: tan 3x tan 2x tan x = tan 3x – tan 2 x – tan x

17. Prove that: $\dfrac{\sec 8A - 1}{\sec 4A - 1} = \dfrac{\tan 8A}{\tan 2A}$

<div align="center">**OR**</div>

Prove that: a(sin B – sin C)+ b(sin C – sin A)+ c(sin A – sin B) = 0.

18. Let A = {a, e, i, o, u} and B = {a, i, k, u}. Find A – B and B – A. Are the two sets A – B and B – A (i) equal (ii) mutually disjoint. Justify your answer.

19. Show that $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta, \; 0<\theta<\dfrac{\pi}{8}$

20. Draw the graph of f(x) $\begin{cases} 3-x, x>1 \\ 1, \quad x=1 \\ 2x, \quad x<1 \end{cases}$ and find the range of f .

<div align="center">**OR**</div>

Draw the graph of f(x) $\begin{cases} 1, \quad x\geq 1 \\ x \quad -1<x<1 \\ -1, \quad x\leq -1 \end{cases}$ `˙˜` d the range of f .

21. Let A = {a, b, c, d} and B = {p, q, r}. Write an example of onto and into function from A to B. Does there exist a one-one function from A to B. Justify your answer.

22. Prove that $\cos^2 x + \cos^2(x+\dfrac{\pi}{3}) + \cos^2(x-\dfrac{\pi}{3}) = \dfrac{3}{2}$

23. The mean of 8, 6, 7, 5, x and 4 is 7. Find (i) the value of x (ii) the mean if each observation was multiplied by 3 (iii) the mean deviation about the median for the original data

<div align="center">**SECTION – D**</div>

24. When two dice are thrown simultaneously, find the probability that neither a doublet nor a total of 10 will appear.

<div align="center">**OR**</div>

The probability that a student will receive A, B, C and D grade are 0.4, 0.35, 0.15 and 0.10 respectively. Find the probability that a student will receive
  1. B or C grade
  2. At most C grade

25. Using mathematical induction prove the following:

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+........+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$

26. (i)A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that
(a) all will be blue? (b) at least one will be green?
(ii) A die has two faces each with number '1', three faces each with number '2' and one face with number '3'. If die is rolled once, determine
    (a)P(2)   (b) P(1 or 3)   (c) P(not 3)

27. Find the derivative of
(i)sin(x + 1) by the abinitio method

(ii)$\dfrac{x}{1 + \tan x}$

OR

Evaluate the limits of the following two functions of x:

(i)$\lim\limits_{x\to 0}\left[\dfrac{x-2}{x^2-x}-\dfrac{1}{x^3-3x^2+2x}\right]$

(ii)$\lim\limits_{x\to 0}\dfrac{\sin 4x}{\sin 2x}$

28. Find the length of the perpendicular drawn from the points,

$(\sqrt{a^2-b^2},0)$ and $(-\sqrt{a^2-b^2},0)$ to the line $\dfrac{x}{a}\cos\theta+\dfrac{y}{b}\sin\theta=1$

Show that their product is b².

29. Solve the inequalities and represent the solution graphically

$5(2x-7)-3(2x+3)\le 0;\ 2x+19\le 6x+47$ and $7\le\dfrac{(3x+11)}{2}\le 11$

OR

How many litres of water will have to be added to 1125 litres of a 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

SECTION – A

1. We have,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{2}{\frac{2}{3}} = \frac{3}{\sin B}$$

$$\Rightarrow 3 = 3\sin B$$

$$\Rightarrow \sin B = 1$$

$$\Rightarrow m\angle B = 90°$$

2.

$$\left(\frac{1}{i}\right)^{25} = \left(\frac{1}{i} \times \frac{i}{i}\right)^{25} = \left(\frac{1}{i^2}\right)^{25}(i)^{25} = (-1)(i)^{25} = (-1)(i)^{24}i = (-1)\left(i^2\right)^{12}i$$

$$= (-1)(-1)^{12}i = (-1)(1)i = -i$$

OR

(1 + i)(1 + 2i) = 1 + 2i + i + 2i² = 1 + 3i – 2 = -1 + 3i

Comparing with a + bi we get a = -1 and b = 3

3. p: I have the money;

q: I will buy an i-phone,

p → q: If I have the money (→) then I will buy an i-phone

4. Relation R from P to Q is R = {(9, 3), (9, –3), (4, 2), (4, –2), (25, 5), (25, –5)}

SECTION – B

5. According to the question,

n(P) = 40, n(P ∪ Q) = 60 and n(P ∩ Q) = 10

n(P ∪ Q) = n(P) + n(Q) – n(P ∩ Q)

60 = 40 + n(Q) – 10

n(Q) = 60 – 40 + 10 = 30

6. We know that the equation of the circle described on the line segment joining (x_1, y_1) and (x_2, y_2) as a diameter is,

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Here, $x_1 = -1$, $x_2 = 4$, $y_1 = 2$ and $y_2 = -3$

Thus, the equation of the required circle is

$(x + 1)(x - 4) + (y - 2)(y + 3) = 0$

$\Rightarrow x^2 + y^2 - 3x + y - 10 = 0$

7. One ace can be selected from 4 aces in $^4C_1$.

Other 4 cards which are non-aces can be selected out of 48 cards in $^{48}C_4$ ways.

The total number of ways = $^4C_1$ x $^{48}C_4$

= 4 x 2 x 47 x 46 x 45 = 778320

OR

$S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

    1. A : Getting a prime number

       $A = \{2, 3, 5\}$

       $n(A) = 3$

       Required probability = 3/6 = ½

    2. B : A number more tan 6

       As B is null set.

       $P(B) = 0$

8. Let ,

$\cos 5x = (\cos 5x + \cos x) - \cos x$

$= 2\cos\dfrac{5x + x}{2}\cos\dfrac{5x - x}{2} - \cos x$

$= 2\cos 3x \cos 2x - \cos x$

$= 2(4\cos^3 x - 3\cos x)(2\cos^2 x - 1) - \cos x$

$= 16\cos^5 x - 20\cos^3 x + 5\cos x$

9. 6 beads have to be arranged in a circular fashion which can be done in (6-1)!

But anticlockwise and clockwise arrangement of beads in a necklace are same so

$(6-1)! \times (1/2) = 60$

OR

$$\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!}$$

$$\frac{1}{9!}+\frac{1}{10\times 9!}=\frac{x}{11\times 10\times 9!}$$

$$\frac{1}{9!}\left(1+\frac{1}{10}\right)=\left(\frac{x}{11\times 10}\right)\frac{1}{9!}$$

$$\frac{11}{10}=\frac{x}{11\times 10}$$

$$x=121$$

10.

Given that the first term is 1.

Also given that each term is the sum of all the terms which follow it.

Let 1, r, r², ... be an infinite G.P., where r is the common ratio.

Sum of terms of an infinite G.P., $S=\dfrac{a}{1-r}$

Here, a = r

Thus, $S=\dfrac{r}{1-r}$

From the given statement of the problem, we have,

$$1=\frac{r}{1-r}$$

$$\Rightarrow 1-r=r$$

$$\Rightarrow r+r=1$$

$$\Rightarrow 2r=1$$

$$\Rightarrow r=\frac{1}{2}$$

Thus the required G.P. is:

$$1,\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16},...$$

11. $-\dfrac{2}{7},k,-\dfrac{7}{2}$ are in G.P.

$$\Rightarrow k^2=\left(-\frac{2}{7}\right)\times\left(-\frac{7}{2}\right)=1$$

$$\Rightarrow k^2=1$$

$$\Rightarrow k=\pm 1$$

When $k=1$; G.P. : $-\dfrac{2}{7},1,-\dfrac{7}{2}$

$r = \dfrac{\dfrac{1}{2}}{-\dfrac{2}{7}} = -\dfrac{7}{2}$

When $k = -1$; GP : $-\dfrac{2}{7}, -1, -\dfrac{7}{2}$

$r = \dfrac{-1}{-\dfrac{2}{7}} = \dfrac{7}{2}$

OR

3, 6, 12,...

a = 3, r = 2 and n = 7

$S_7 = a\left(\dfrac{r^7 - 1}{r - 1}\right) = 3\left(\dfrac{2^7 - 1}{2 - 1}\right) = 3 \times 127 = 381$

12. $a_n = \dfrac{(1 + 2 + 3 + \ldots n)}{n} = \dfrac{n(n+1)}{2n}$

$S_n = \sum_n a_n$

$= \dfrac{1}{2} \sum_{i=1}^{n} (n+1)$

$= \dfrac{1}{2} \dfrac{n(n+1)}{2} + \dfrac{n}{2}$

$= \dfrac{(n^2 + n)}{4} + \dfrac{n}{2}$

$= \dfrac{n(n+3)}{4}$

SECTION – C

13.

$\dfrac{1+i}{1-i} - \dfrac{1-i}{1+i} = \dfrac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)}$

$= \dfrac{\left(1 + i^2 + 2i\right) - \left(1 + i^2 - 2i\right)}{(1-i)(1+i)}$

$= \dfrac{(1 - 1 + 2i) - (1 - 1 - 2i)}{\left(1 - i^2\right)} = \dfrac{(2i) - (-2i)}{\left(1 - i^2\right)}$

$$= \frac{2i + 2i}{(1 - (-1))} = \frac{4i}{(2)} = 2i$$

$$\left| \frac{1+i}{1-i} - \frac{1-i}{1+i} \right| = |2i| = \sqrt{0^2 + 2^2} = \sqrt{4} = 2$$

OR

Consider the given equation:

(a + ib) (c + id) (e + if) (g + ih) = A + iB

Let us take modulus on both sides ,

$|(a + ib) (c + id) (e + if) (g + ih)| = |A + iB|$

We know, $|z_1 z_2| = |z_1||z_2|$

$\therefore |(a + ib) (c + id) (e + if) (g + ih)| = |A + iB|$

$\Rightarrow |(a + ib)||(c + id)||(e + if)||(g + ih)| = |A + iB|$

$\Rightarrow \sqrt{a^2 + b^2}.\sqrt{c^2 + d^2}.\sqrt{e^2 + f^2}.\sqrt{g^2 + h^2} = \sqrt{A^2 + B^2}$

$\Rightarrow \left[\sqrt{a^2 + b^2}.\sqrt{c^2 + d^2}.\sqrt{e^2 + f^2}.\sqrt{g^2 + h^2} \right]^2 = \left[\sqrt{A^2 + B^2} \right]^2$

$\Rightarrow (a^2 + b^2).(c^2 + d^2).(e^2 + f^2).(g^2 + h^2) = A^2 + B^2$

Thus, the value of $A^2 + B^2 = (a^2 + b^2).(c^2 + d^2).(e^2 + f^2).(g^2 + h^2)$

14. Consider the given quadratic equation:

$9x^2 - 12x + 20 = 0$

$\Rightarrow 3x^2 - 4x + \frac{20}{3} = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 3 \times \frac{20}{3}}}{2.3} = \frac{4 \pm \sqrt{16 - 4 \times 20}}{6}$

$= \frac{4 \pm \sqrt{16 - 80}}{6} = \frac{4 \pm \sqrt{-64}}{6} = \frac{4 \pm 8\sqrt{-1}}{6} = \frac{2 \pm 4i}{3}$

$\Rightarrow x = \frac{2}{3} \pm \frac{4}{3}i$

15. To find the coefficient of x^5 in the expansion of the product,

$(1 + 2x)^6(1-x)^7$

Let us find the expansions of the 2 binomials.

$$(1 + 2x)^6 = {}^6C_0(2x)^0 + {}^6C_1(2x)^1 + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4$$
$$+ {}^6C_5(2x)^5 + {}^6C_6(2x)^6$$
$$= 1 \times 1 + 6 \times (2x) + 15 \times (2x)^2 + 20 \times (2x)^3 + 15 \times (2x)^4$$
$$+ 6 \times (2x)^5 + 1 \times (2x)^6$$
$$= 1 + 12x + 60x^2 + 20 \times (2x)^3 + 15 \times (2x)^4$$
$$+ 6 \times (2x)^5 + 1 \times (2x)^6$$

$$(1-x)^7 = {}^7C_0(-x)^0 + {}^7C_1(-x)^1 + {}^7C_2(-x)^2 + {}^7C_3(-x)^3 + {}^7C_4(-x)^4$$
$$+ {}^7C_5(-x)^5 + {}^7C_6(-x)^6 + {}^7C_7(-x)^7$$
$$= 1 \times 1 - 7 \times (x) + 21 \times (x)^2 - 35 \times (x)^3 + 35 \times (x)^4$$
$$- 21 \times (x)^5 + 7 \times (x)^6 - 1 \times (x)^7$$
$$= 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7$$

Product of $(1 + 2x)^6(1-x)^7$

$$= \left[\boxed{1} + \boxed{12}x + \boxed{60}x^2 + \boxed{160}x^3 + \boxed{240}x^4 + \boxed{192}x^5 + 64x^6 \right]$$
$$\times \left[\boxed{1}\,\boxed{-7}x + \boxed{21}x^2\,\boxed{-35}x^3 + \boxed{35}x^4\,\boxed{-21}x^5 + 7x^6 - x^7 \right]$$

$\therefore$ Coefficient of x^5 in the product

$$= 1 \times (-21) + 12 \times 35 + 60 \times (-35) + 160 \times 21 + 240 \times (-7) + 192 \times 1$$
$$= -21 + 420 - 2100 + 3360 - 1680 + 192$$
$$= 3972 - 3801$$
$$= 171$$

16. Consider, $3x = 2x + x$

Rewriting $\tan 3x = \tan(2x + x)$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

$\tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$

or $\tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$

or $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$

17. We need to prove that,

$$\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$$

Consider L.H.S.

$$\frac{\dfrac{1-\cos 8A}{\cos 8A}}{\dfrac{1-\cos 4a}{\cos 4a}} = \frac{2\sin^2 4A}{\cos 8A} \times \frac{\cos 4A}{2\sin^2 2A}$$

$$= \frac{2\sin 4A \times \cos 4A \times \sin 4A}{\cos 8A \times 2\sin 2A \times \sin 2A}$$

$$= \frac{\sin 8A}{\cos 8A} \times \frac{2\sin 2A \times \cos 2A}{2\sin 2A \times \sin 2A} = \frac{\tan 8A}{\tan 2A}$$

OR

Consider the L.H.S of the given equation:

$$\text{L.H.S} = a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B)$$

Let us use the identity,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Thus, $\text{L.H.S} = a\sin B - a\sin C + b\sin C - b\sin A + c\sin A - c\sin B$

$\Rightarrow \text{L.H.S} = b\sin A - c\sin A + c\sin B - b\sin A + c\sin A - c\sin B$
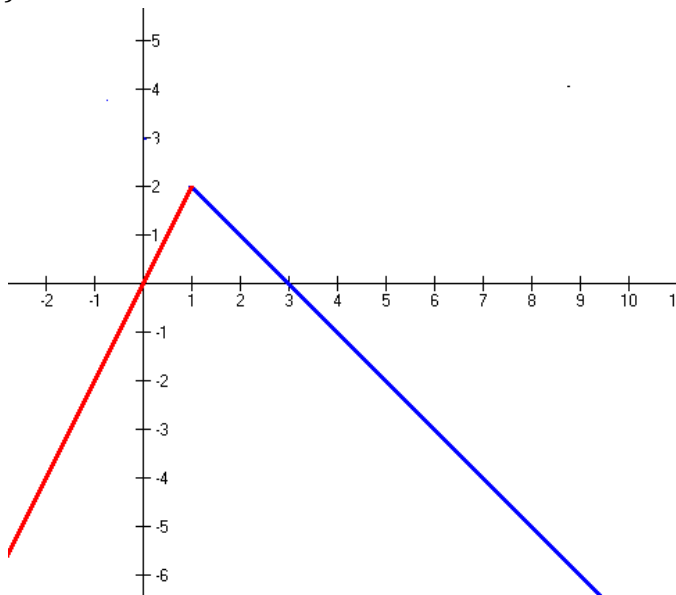
$\Rightarrow \text{L.H.S} = 0$

R.H.S = 0

Hence proved.

18. A – B = {*e, o*}, since the elements *e, o* belong to A but not to B and B – A = {*k*}, since the element *k* belongs to B but not to A.
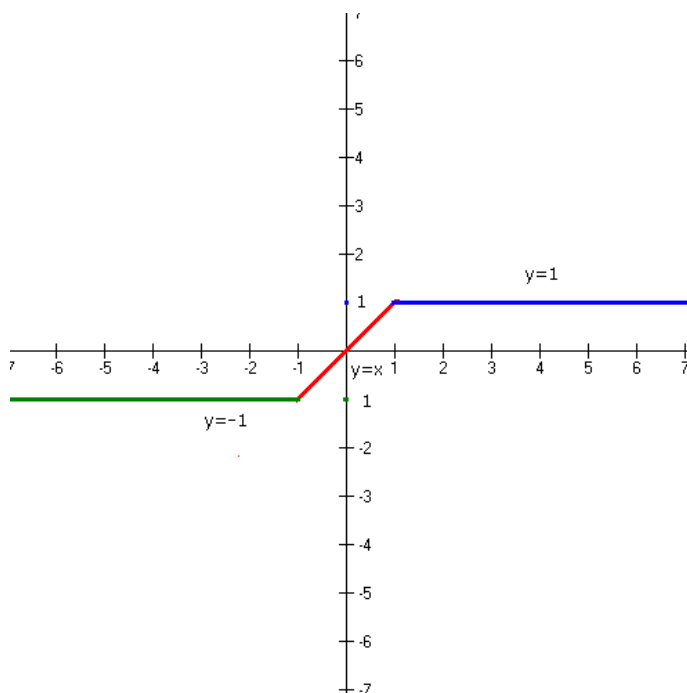
(i) We note that A – B ≠ B – A.

(ii) The sets (A – B) and (B – A) are mutually disjoint sets, i.e. the intersection of these two sets is a null set.

19. $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}$

$=\sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8\theta)}}}$

$=\sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2 4\theta}}}$

$=\sqrt{2+\sqrt{2+2\cos 4\theta}}$

$=\sqrt{2+\sqrt{2(1+\cos 4\theta)}}$

$=\sqrt{2+\sqrt{2\times 2\cos^2 2\theta}}$

$=\sqrt{2+2\cos 2\theta}$

$=\sqrt{2(1+\cos 2\theta)}$

$=\sqrt{2\times 2\cos^2 \theta}$

$=2\cos\theta$

$\therefore \sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=2\cos\theta$

20. Range of $f=(-\infty, 2)$

OR

Range f = [-1, 1]

21. Into Function: {a, p), (b, q), (c,p),(d,p)} range must be the proper subset of set B
    Onto: {a, p), (b, q) (c, r), (d, r)} range must be same as set B
    No one-one function can be defined from A to B because n(B) < n(A)

22. $\cos^2 x + \cos^2(x + \frac{\pi}{3}) + \cos^2(x - \frac{\pi}{3})$

$= \cos^2 x + \cos^2(x + \frac{\pi}{3}) + 1 - \sin^2(x - \frac{\pi}{3})$

$= 1 + \cos^2 x + \left[\cos^2(x + \frac{\pi}{3}) - \sin^2(x - \frac{\pi}{3})\right]$

$= 1 + \cos^2 x + \left[\cos(x + \frac{\pi}{3} + x - \frac{\pi}{3})\cos(x + \frac{\pi}{3} - x + \frac{\pi}{3})\right] \quad \left[\because \cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)\right]$

$= 1 + \cos^2 x + \cos(2x)\cos\left(\frac{2\pi}{3}\right)$

$= 1 + \cos^2 x + \cos(2x)\left(-\frac{1}{2}\right)$

$= 1 + \cos^2 x + \left(2\cos^2 x - 1\right)\left(-\frac{1}{2}\right)$

$= 1 + \cos^2 x + \left(-\cos^2 x + \frac{1}{2}\right)$

$= 1 + \frac{1}{2} = \frac{3}{2}$

23. (i)

$$7 = \frac{8+6+7+5+x+4}{6}$$
$$\Rightarrow 42 = 8+6+7+5+x+4$$
$$\Rightarrow 42 = 30+x$$
$$\Rightarrow x = 12$$

(ii) If each observation is multiplied by 3, the mean will also be multiplied by 3, hence the mean is 21.

(iii) Arranging in the ascending order 4, 5, 6, 7, 8, 12

Median = $\frac{6+7}{2} = 6.5$ ....

Forming the table, we get

x_i	4	5	6	7	8	12
$x_{i-6.5}$	-2.5	-1.5	-0.5	0.5	1.5	5.5
$|x_i - 6.5|$	2.5	1.5	0.5	0.5	1.5	5.5

$$\sum_{i=1}^{n} |x_i - M| = \sum_{i=1}^{n} |x_i - 6.5| = 12$$

$$\text{M.D(M)} = \frac{\sum_{i=1}^{n} |x_i - M|}{n} = \frac{\sum_{i=1}^{n} |x_i - 6.5|}{6}$$

$$= \frac{12}{6} = 2.0$$

SECTION – D

24. Let S be the sample space. Then n(S) = 36

Let E_1 = event that a doublet appears

Let E_2 = event of getting a total of 10.

Then, $E_1 = \big[(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\big]$,

and $E_2 = \big[(4,6),(5,5),(6,4)\big]$

$\therefore E_1 \cap E_2 = \big[(5,5)\big]$

So, $n(E_1) = 6, n(E_2) = 3$ and $n(E_1 \cap E_2) = 1$.

Thus,

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6},$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

$$P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{1}{36}$$

Therefore, the probability of getting a doublet or a total of 10

$$= P(E_1 \text{ or } E_2)$$

$$= P(E_1 \cup E_2)$$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{1}{6} + \frac{1}{12} - \frac{1}{36}$$

$$= \frac{8}{36}$$

$$= \frac{2}{9}$$

Thus P(getting neither a doublet nor a total of 10)

$$= P\left(\overline{E_1} \text{ and } \overline{E_2}\right)$$

$$= P\left(\overline{E_1} \cap \overline{E_2}\right)$$

$$= P\left(\overline{E_1 \cup E_2}\right)$$

$$= 1 - P(E_1 \cup E_2)$$

$$= 1 - \frac{2}{9}$$

$$= \frac{7}{9}$$

OR

Let E_1, E_2, E_3 and E_4 denote respectively the events that a student will receive A, B, C and D grades.

$P(E_1) = 0.4$, $P(E_2) = 0.35$, $P(E_3) = 0.15$ and $P(E_4) = 0.10$

1. Required probability = $P(E_2 \cup E_3)$
   = $P(E_2) + P(E_3)$ $\because$ Mutually exclusive events
   = 0.35 + 0.15 = 0.5

2. Required probability = $P(E_3 \cup E_4)$
   = $P(E_3) + P(E_4)$ $\because$ Mutually exclusive events
   = 0.15 + 0.1
   = 0.25

25. Let the statement P(n) be: $\dfrac{1}{1.2.3} + \dfrac{1}{2.3.4} + \dfrac{1}{3.4.5} + \ldots + \dfrac{1}{(n)(n+1)(n+2)} = \dfrac{(n)(n+3)}{4(n+1)(n+2)}$

Consider P(1): $\dfrac{1}{1.2.3} = \dfrac{(1)(1+3)}{4(1+1)(1+2)}$

$\Rightarrow \dfrac{1}{1.2.3} = \dfrac{(1)(1+3)}{4(1+1)(1+2)} = \dfrac{1.4}{4.2.3} = \dfrac{1}{1.2.3}$ [P(1) is true]

Let us assume that P(k) is true

P(k): $\dfrac{1}{1.2.3} + \dfrac{1}{2.3.4} + \dfrac{1}{3.4.5} + \ldots + \dfrac{1}{(k)(k+1)(k+2)} = \dfrac{(k)(k+3)}{4(k+1)(k+2)}$

To prove:

P(k+1): $\dfrac{1}{1.2.3} + \dfrac{1}{2.3.4} + \dfrac{1}{3.4.5} + \ldots + \dfrac{1}{(k)(k+1)(k+2)} + \dfrac{1}{(k+1)(k+2)(k+3)}$

$= \dfrac{(k+1)(k+4)}{4(k+2)(k+3)}$

LHS $= \dfrac{1}{1.2.3} + \dfrac{1}{2.3.4} + \dfrac{1}{3.4.5} + \ldots + \dfrac{1}{(k)(k+1)(k+2)} + \dfrac{1}{(k+1)(k+2)(k+3)}$

$= \dfrac{1}{1.2.3} + \dfrac{1}{2.3.4} + \dfrac{1}{3.4.5} + \ldots + \dfrac{1}{k(k+1)(k+2)} + \dfrac{1}{(k+1)(k+2)(k+3)}$

$= \dfrac{k(k+3)}{4(k+1)(k+2)} + \dfrac{1}{(k+1)(k+2)(k+3)}$ (using P(k))

$= \dfrac{1}{(k+1)(k+2)}\left[\dfrac{k(k+3)}{4} + \dfrac{1}{(k+3)}\right]$

$= \dfrac{1}{(k+1)(k+2)}\left[\dfrac{k(k+3)^2 + 4}{4(k+3)}\right]$

$= \dfrac{1}{(k+1)(k+2)}\left[\dfrac{k(k^2 + 9 + 6k) + 4}{4(k+3)}\right]$

$= \dfrac{1}{(k+1)(k+2)}\left[\dfrac{k^3 + 9k + 6k^2 + 4}{4(k+3)}\right]$

$= \dfrac{1}{(k+1)(k+2)}\left[\dfrac{(k+1)^2(k+4)}{4(k+3)}\right]$

$= \dfrac{(k+1)(k+4)}{4(k+2)(k+3)} = $ R.H.S.

26.

(i) There are a total of 60 marbles out of which 5 marbles are to be selected

Number of ways in which 5 marbles are to be selected out of 60 = $^{60}C_5$

(a)

Out of 20 blue marbles, 5 can be selected in = $^{20}C_5$

$$P(\text{all 5 blue marbles}) = \frac{^{20}C_5}{^{60}C_5} = \frac{\dfrac{20!}{5!15!}}{\dfrac{60!}{5!55!}} = \frac{20!5!55!}{5!15!60!}$$

$$= \frac{20.19.18.17.16}{60.59.58.57.56} = \frac{34}{11977}$$

(b) Number of ways in which 5 marbles are to be selected out of 60 = $^{60}C_5$.

Out of 30 non-green marbles, 5 can be selected in $^{30}C_5$

$$P(\text{all 5 non-green marbles}) = \frac{^{30}C_5}{^{60}C_5} = \frac{\dfrac{30!}{5!25!}}{\dfrac{60!}{5!55!}} = \frac{30!5!55!}{5!25!60!}$$

$$= \frac{30.29.28.27.26.}{60.59.58.57.56} = \frac{117}{4484}$$

$$P(\text{atleast one green marble}) = 1 - P(\text{all 5 non} - \text{green marbles})$$

$$= 1 - \frac{117}{4484}$$

$$= \frac{4484 - 117}{4484} = \frac{4367}{4484}$$

(ii) On the dice two faces are with number '1', three faces are with
number '2' and one face is with number '3'

$$\therefore P(1) = \frac{2}{6} = \frac{1}{3}; P(2) = \frac{3}{6} = \frac{1}{2}; P(3) = \frac{1}{6}$$

$$\therefore (i)\ P(2) = \frac{1}{2}$$

(ii) $P(1 \text{ or } 3) = P(1) + P(3)$ [The events are mutually exclusive]

$$= \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

(iii) $P(\text{not } 3) = 1 - P(3) = 1 - \frac{1}{6} = \frac{5}{6}$

27. (i) sin(x+1) by the definition method

Let $y = \sin(x+1)$

$\Rightarrow y + \delta y = \sin(x+\delta x+1)$

$\Rightarrow \delta y = (y+\delta y) - y = \sin(x+\delta x+1) - \sin(x+1)$

$= 2\cos(\dfrac{x+\delta x+1+x+1}{2})\sin(\dfrac{x+\delta x+1-(x+1)}{2})$

$= 2\cos(\dfrac{2x+\delta x+2}{2})\sin(\dfrac{\delta x}{2})$

$\dfrac{\delta y}{\delta x} = \dfrac{1}{\delta x}2\cos(\dfrac{2x+\delta x+2}{2})\sin(\dfrac{\delta x}{2})$

$\Rightarrow \dfrac{\delta y}{\delta x} = \cos(\dfrac{2x+\delta x+2}{2})\dfrac{\sin(\dfrac{\delta x}{2})}{\dfrac{\delta x}{2}}$

$\Rightarrow \lim_{\delta x \to 0}\dfrac{\delta y}{\delta x} = \lim_{\delta x \to 0}\cos(\dfrac{2x+\delta x+2}{2})\dfrac{\sin(\dfrac{\delta x}{2})}{\dfrac{\delta x}{2}}$

$\Rightarrow \lim_{\delta x \to 0}\dfrac{\delta y}{\delta x} = \lim_{\delta x \to 0}\cos(\dfrac{2x+\delta x+2}{2})\lim_{\delta x \to 0}\dfrac{\sin(\dfrac{\delta x}{2})}{\dfrac{\delta x}{2}}$

$\Rightarrow \dfrac{dy}{dx} = \cos(\dfrac{2x+2}{2}).1$

$\Rightarrow \dfrac{dy}{dx} = \cos(x+1)$

(ii) Let $y = \dfrac{x}{1+\tan x}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{(1+\tan x)\dfrac{d}{dx}(x) - (x)\dfrac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{(1+\tan x).1 - (x)(\sec^2 x)}{(1+\tan x)^2}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1+\tan x - x\sec^2 x}{(1+\tan x)^2}$

$$\left[\frac{x-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right]=\left[\frac{x-2}{x(x-1)}-\frac{1}{x(x^2-3x+2)}\right]$$

$$=\left[\frac{x-2}{x(x-1)}-\frac{1}{x(x-1)(x-2)}\right]$$

$$=\left[\frac{x^2-4x+4-1}{x(x-1)(x-2)}\right]$$

$$=\frac{x^2-4x+3}{x(x-1)(x-2)}$$

$$\lim_{x\to1}\left[\frac{x^2-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right]=\lim_{x\to1}\frac{x^2-4x+3}{x(x-1)(X-2)}$$

$$=\lim_{x\to1}\frac{(x-3)(x-1)}{x(x-1)(x-2)}$$

$$=\lim_{x\to1}\frac{x-3}{x(x-2)}=\frac{1-3}{1(1-2)}=2$$

(ii) $\lim_{x\to0}\dfrac{\sin4x}{\sin2x}=\lim_{x\to0}\left[\dfrac{\sin4x}{4x}\times\dfrac{2x}{\sin2x}\times2\right]$

$$=2\times\lim_{x\to0}\left[\frac{\sin4x}{4x}\right]\div\left[\frac{\sin2x}{2x}\right]$$

$$=2\times\lim_{4x\to0}\left[\frac{\sin4x}{4x}\right]\div\lim_{2x\to0}\left[\frac{\sin2x}{2x}\right]$$

$$=2\times1\times1=2 \qquad (\text{as } x\to0, 4x\to0 \text{ and } 2x\to0)$$

28. Perpendicular distance of the point (m,n) from the line ax+by+c=0, is given by

$$\left| \frac{a(m) + b(n) + c}{\sqrt{a^2 + b^2}} \right|$$

∴ Perpendicular distance of the point $(\sqrt{a^2 - b^2}, 0)$ from the line

$\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta - 1 = 0$, is given by

$$\left| \frac{\dfrac{\cos\theta}{a}(\sqrt{a^2 - b^2}) + \dfrac{\sin\theta}{b}(0) - 1}{\sqrt{\left(\dfrac{\cos\theta}{a}\right)^2 + \left(\dfrac{\sin\theta}{b}\right)^2}} \right| = \left| \frac{\dfrac{\cos\theta}{a}(\sqrt{a^2 - b^2}) - 1}{\sqrt{\left(\dfrac{\cos\theta}{a}\right)^2 + \left(\dfrac{\sin\theta}{b}\right)^2}} \right|$$

Perpendicular distance of the point $(-\sqrt{a^2 - b^2}, 0)$ from the line

$\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta - 1 = 0$, is given by

$$\left| \frac{-\dfrac{\cos\theta}{a}(\sqrt{a^2 - b^2}) + \dfrac{\sin\theta}{b}(0) - 1}{\sqrt{\left(\dfrac{\cos\theta}{a}\right)^2 + \left(\dfrac{\sin\theta}{b}\right)^2}} \right| = \left| \frac{-\dfrac{\cos\theta}{a}(\sqrt{a^2 - b^2}) - 1}{\sqrt{\left(\dfrac{\cos\theta}{a}\right)^2 + \left(\dfrac{\sin\theta}{b}\right)^2}} \right|$$

Product of the perpendicular distances

$$= \left| \frac{\dfrac{\cos\theta}{a}(\sqrt{a^2 - b^2}) - 1}{\sqrt{\left(\dfrac{\cos\theta}{a}\right)^2 + \left(\dfrac{\sin\theta}{b}\right)^2}} \right| \times \left| \frac{-\dfrac{\cos\theta}{a}(\sqrt{a^2 - b^2}) - 1}{\sqrt{\left(\dfrac{\cos\theta}{a}\right)^2 + \left(\dfrac{\sin\theta}{b}\right)^2}} \right|$$

$$= \left| \frac{\left[\dfrac{\cos\theta}{a}(\sqrt{a^2 - b^2}) - 1\right]\left[\dfrac{\cos\theta}{a}(\sqrt{a^2 - b^2}) + 1\right]}{\sqrt{\left(\dfrac{\cos\theta}{a}\right)^2 + \left(\dfrac{\sin\theta}{b}\right)^2} \sqrt{\left(\dfrac{\cos\theta}{a}\right)^2 + \left(\dfrac{\sin\theta}{b}\right)^2}} \right|$$

$$= \left| \frac{\left[\dfrac{(a^2 - b^2)\cos^2\theta}{a^2} - 1\right]}{\left(\dfrac{\cos\theta}{a}\right)^2 + \left(\dfrac{\sin\theta}{b}\right)^2} \right|$$

$$= \frac{\left\|\left[\dfrac{\left(a^2 - b^2\right)\cos^2\theta}{a^2} - 1\right]\right\|}{\dfrac{b^2\cos^2\theta + a^2\sin^2\theta}{a^2 b^2}}$$

$$= \frac{a^2 b^2 \left\|\left[\dfrac{a^2\cos^2\theta - b^2\cos^2\theta - a^2}{a^2}\right]\right\|}{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$= \frac{b^2 \left|a^2\cos^2\theta - b^2\cos^2\theta - a^2\right|}{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$= \frac{b^2 \left|-a^2(1 - \cos^2\theta) - b^2\cos^2\theta\right|}{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$= \frac{b^2 \left|-a^2(\sin^2\theta) - b^2\cos^2\theta\right|}{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$= \frac{b^2 \left|-\left(a^2(\sin^2\theta) + b^2\cos^2\theta\right)\right|}{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$= \frac{b^2 \left(a^2(\sin^2\theta) + b^2\cos^2\theta\right)}{b^2\cos^2\theta + a^2\sin^2\theta}$$

$$= b^2$$

29. $5(2x - 7) - 3(2x + 3) \leq 0$;

$2x + 19 \leq 6x + 47$

and

$7 \leq \dfrac{(3x+11)}{2} \leq 11$

Let us solve the inequalities one by one and then work out the common solution.

Inequality 1:

$5(2x - 7) - 3(2x + 3) \leq 0$

$\Rightarrow 10x - 35 - 6x - 9 \leq 0$

$\Rightarrow 4x - 44 \leq 0$

$\Rightarrow 4x \leq 44$

$\Rightarrow x \leq 11$

Inequality 2:

$2x + 19 \leq 6x + 47$

$\Rightarrow 2x - 6x \leq -19 + 47$

$\Rightarrow -4x \leq 28$

$\Rightarrow -x \leq 7$

$\Rightarrow x \geq -7$
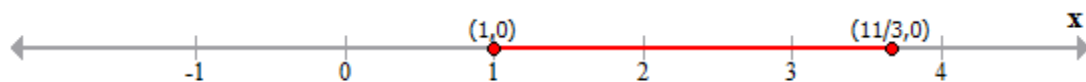
Inequality 3:

$7 \leq \dfrac{(3x+11)}{2} \leq 11$

$\Rightarrow 14 \leq 3x + 11 \leq 22$

$\Rightarrow 14 - 11 \leq 3x \leq 22 - 11$

$\Rightarrow 3 \leq 3x \leq 11$

$\Rightarrow 1 \leq x \leq \dfrac{11}{3}$

$x \leq 11, x \geq -7, 1 \leq x \leq \dfrac{11}{3}$ together $\Rightarrow 1 \leq x \leq \dfrac{11}{3}$

The amount of acid in 1125lt of the 45% solution=45% of 1125=$\dfrac{45 \times 1125}{100}$

Let x lt of the water be added to it to obtain a solution between 25% and 30% solution

$$\Rightarrow 25\% < \dfrac{1125 \times \dfrac{45}{100}}{1125 + x} < 30\%$$

$$\Rightarrow \dfrac{25}{100} < \dfrac{1125 \times \dfrac{45}{100}}{1125 + x} < \dfrac{30}{100}$$

$$\Rightarrow \dfrac{25}{100} < \dfrac{1125 \times 45}{(1125 + x) \times 100} < \dfrac{30}{100}$$

$$\Rightarrow 25 < \dfrac{1125 \times 45}{(1125 + x)} < 30$$

$$\Rightarrow \dfrac{1}{25} > \dfrac{(1125 + x)}{1125 \times 45} > \dfrac{1}{30}$$

$$\Rightarrow \dfrac{1125 \times 45}{25} > (1125 + x) > \dfrac{1125 \times 45}{30}$$

$$\Rightarrow \dfrac{50625}{25} > (1125 + x) > \dfrac{50625}{30}$$

$$\Rightarrow 2025 > (1125 + x) > 1687.5$$

$$\Rightarrow 2025 > (1125 + x) > 1687.5$$

$$\Rightarrow 2025 - 1125 > x > 1687.5 - 1125$$

$$\Rightarrow 900 > x > 562.5$$

$$\Rightarrow 562.5 < x < 900$$

So the amount of water to be added must be between 562.5 to 900 lt